SMASK RECESS TERM S.6 PURE MTC ASSIGNMENT

ALGEBRA

1. Show that, if $x^2+x+b=0$ and $x^2+ax+b=0$ have a common root then $(b-1)^2=(a-1)(1-ab)$.
2. If the equations $x^2+ax+b=0$ and $cx^2+2ax-3b=0$ have a common root, prove that $b=\frac{5a^2(c-2)}{(c+3)^2}$.
3. Find the condition for the equations $x^2+2x+a=0$ and $x^2+bx+3=0$ to have a common root.
4. Solve the equations;
   (i) $2-5e^{-x}+5e^{-2x}=0$.
   (ii) $x^2+2x=34+\frac{35}{x^2+2x}$.
   (iii) $x^{\frac{4}{3}}+16x^{\frac{-4}{3}}=17$.
   (iv) $2x^4-9x^3+14x^2-9x+2=0$.
   (v) $4x^2+25y^2=100; xy=4$.
   (vi) $9x^{\frac{2}{3}}+4x^{\frac{-2}{3}}=37$.
   (vii) $\frac{1}{x}-\frac{1}{x+3}=\frac{1}{k}-\frac{1}{k+3}$.
   (viii) $\sqrt{2-x}+\sqrt{3+x}=3$.
5. Use row reduction to solve the simultaneous equations $2x+3y+4z=8, 3x-2y-3z=-2, 5x+4y+2z=3$.
6. Use row reduction to Achelon form to solve $2x+3y+4z=8, 3x-2y-3z=-2, 5x+4y+2z=3$.
7. Write down the sum, the sum of the product in pairs and the product of the roots of the equations;
   (i) $3x^3-4x^2+2x+5=0$.
   (ii) $x^3-5x^2+2=0$.

   HINT: If the roots of the equation $ax^3+bx^2+cx+d=0$ are α,β,γ then
   $\alpha+\beta+\gamma=-\frac{b}{a}; \alpha\beta+\alpha\gamma+\gamma\beta=\frac{c}{a}; \alpha\beta\gamma=\frac{d}{a}$.
8. Find the relationship between a, b and c if one root of the equation $ax^3+bx^2+cx+d=0$ is equal to the sum of the other two.
9. The roots of the equation $x^3-5x^2+x+12=0$ are α,β,γ. Calculate the value of $(\alpha+2)(\beta+2)(\gamma+2)$.
10. If α is a repeated root of the equation $f(x)=0$, prove that α is also a root of the equation $f^1(x)=0$. Hence solve the equation $18x^3+3x^2-88x-80=0$ given that it has a repeated root.
11. If $(x+1)^2$ is a factor of $2x^4+7x^3+6x^2+Ax+B$, find the values of A and B.
12. Solve the equation $5x^3-111x^2+74x-16=0$ given that the roots are in a G.P.
13. Solve the equation $64x^3-240x^2+284x-105=0$ given that the roots are in A.P.
14. Solve the equation $3x^3+14x^2+2x-4=0$.
15. Prove that the remainder when $p(x)$ is divided by $(x-a)^2$ is $(x-a)p^1(a)+p(a)$. Hence given that x^4+bx+c is divisible $(x-2)^2$, find the value of b and c.
16. If the roots of the equation $x^3-5x^2+qx-8=0$ are in G.P, show that $q=10$.
17. Prove that, if two polynomials $p(x)$ and $q(x)$ have a common factor $x-p$, then $x-p$ is a factor of $p(x)-q(x)$. Hence prove that, if the equations $ax^3+4x^2-5x-10=0$ and $ax^3-9x-2=0$ have a common root then $a=2, a=11$.
18. Find the equation of the tangent to the curve $y=x^3$ at (t,t^3) and find the coordinates of the point where the tangent meets the curve again.
19. Given that $12x^3-20x^2-21x+36=0$ has a repeated root, solve the equation.
20. Find the point where tangent at (t^2,t^3) on the curve $y^2=x^3$ meets the curve again.
21. Prove that $\tan 3x=\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$. Hence solve the equation $t^3-6t^2-3t+2=0$ correct to 2 s.f.
22. Prove that $\sin 3\theta=3\sin\theta-4\sin^3\theta$. Hence solve the equation $8x^3-6x+1=0$ correct to 4 s.f.
23. Use the substitution $x=2\sin\theta$, to solve $3x^3-9x+2=0$ correct to 4s.f.
24. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, prove that for any non - zero numbers l, m, n $\frac{la+mc+ne}{lb+md+nf}=\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$.
25. If $\frac{a}{b}=\frac{c}{d}$, prove that $\frac{a-c}{a+c}=\frac{b-d}{b+d}$.
26. If $\frac{a}{b}=\frac{c}{d}$, prove that $\frac{la+mb}{\lambda a+\mu b}=\frac{lc+md}{\lambda c+\mu d}$.
27. If $\frac{a}{b}=\frac{c}{d}$, prove that $\frac{a^2-b^2}{a^2+b^2}=\frac{c^2-d^2}{c^2+d^2}$.
28. Solve the simultaneous equations
    (i) $\frac{x}{1}=\frac{x+y}{3}=\frac{x-y+z}{2}; x^2+y^2+z^2+x+2y+4z-6=0$.
    (ii) $\frac{x-y}{4}=\frac{z-y}{3}=\frac{2z-x}{1}; x+3y+2z=4$.
29. Prove that if $x+\frac{1}{x}=y+1$, then $\frac{(x^2-x+1)^2}{x(x-1)^2}=\frac{y^2}{y-1}$. Hence solve the equation $(x^2-x+1)^2-4x(x-1)^2=0$.
30. Use the remainder theorem to express x^3+2x^2+x-18 as a product of two factors.
31. Find the value of p for which the polynomial $x^4+x^3+px^2+5x-10$ has $x+2$ as a factor.
32. Find the values k and l for which $x^4-2x^3+5x^2+kx+l$ has factor $(x-1)^2$.
33. The roots of the equation $3x^2+kx+12=0$ are equal, find the value of k.
34. Indicate on an Argand diagram the region $0\le\arg(z+2+3i)\le\frac{\pi}{6}$.

35. Show on an Argand diagram the locus of z when

(a) $|z-1-i|=2$

(b) $\operatorname{Re} z = 1$ and $-\frac{\pi}{3} \le \arg z \le \frac{\pi}{4}$. In each case find the least value of $|z|$.

36. If $\arg(z+3)=\frac{\pi}{3}$, find the least value of $|z|$.

37. If $|z-3+2i|=2$, find the greatest and least value of

(i) $|z|$

(ii) $|z+1|$.

38. Prove by induction that $\frac{1}{2}+\frac{1}{6}+\ldots+\frac{1}{n(n-1)}=1-\frac{1}{n}$.

39. Find the sum of integers between 1 and 100 which are not divisible by 3.

40. Solve the inequalities,

(i) $x^4-10x^2+9>0$.

(ii) $\frac{x-1}{x} > \frac{2}{3-x}$

(iii) $\left|\frac{2x-4}{x+1}\right| < 4$

41. Find the range(s) of values of k for which the roots of the equation $(k-2)x^2-(8-2k)x-(8-3k)=0$.

ANAYSIS

1. Integrate the following functions with respect to x;

(a) $\left(x^2-\frac{2}{x}\right)^2$ (b) $\sin 3x \cos 5x$ (c) $x(1+x^2)^{3/2}$ (d) $\frac{1}{\sqrt{5+4x-x^2}}$.

(e) $\frac{\sec^2\sqrt{x}}{\sqrt{x}}$ (f) $\frac{1}{x\sqrt{9x^2-1}}$ (g) $\frac{x+3}{\sqrt{7-6x-x^2}}$

2. Use the substitution $u=+\sqrt{1+x^2}$ to evaluate $\int_{4/3}^{12/5} \frac{dx}{x(1+x^2)^{3/2}}$.

3. Evaluate $\int_1^2 \frac{dx}{x^2\sqrt{x-1}}$, using the substitution $x=\sec^2\theta$.

4. Evaluate $\int_1^2 \frac{dx}{x^2\sqrt{5x^2-1}}$, using (a) $x^2=\frac{1}{u}$ (b) the sine substitution.

5. Use small changes to show that $(16.02)^{\frac{1}{4}} = 2\frac{1}{1600}$.

6. An open cylinder container is made from a $12cm^2$ metal sheet. Show that the maximum volume of the container is $\frac{8}{\sqrt{\pi}}$.

7. Find the area enclosed by the curves $y^2=4x$ and $x^2=4y$.

8. Find the equation of the tangent t the point $(1,-1)$ to the curve $y=2-4x^2+x^3$. What are the coordinates of the point where the tangent meets the curve again? Find the eqution of the tangent at this point.

9. If $y=\tan\left(2\tan^{-1}\frac{x}{2}\right)$, show that $\frac{dy}{dx}=\frac{4(1+y^2)}{4+x^2}$.

10. Differentiate $\sqrt{\cos x}$ from first principles.

11. A particle is moving in a straight line such that its distance from a fixed point O, t seconds after motion begins is $s=\cos t+\cos 2t$. Find

(i) The time when the particle passes through O.

(ii) The velocity of the particle at this instant.

(iii) The acceleration when the velocity is zero.

TRIGONOMETRY

1. Prove that $(\sin 2\alpha - \sin 2\beta)\tan(\alpha+\beta) = 2(\sin^2\alpha - \sin^2\beta)$

2. Given that $\sin(x+\beta)=2\cos(x-\beta)$, prove that $\tan x = \frac{2-\tan\alpha}{1-2\tan\alpha}$.

3. Solve the equation $\cos(2\theta+45^\circ)-\cos(2\theta-45^\circ)=1$ for $0^\circ \le \theta \le 360^\circ$.

4. Prove that $\cos^6 x + \sin^6 x = 1-\frac{3}{4}\sin^2 2x$.

5. The roots of the equation $ax^2+bx+c=0$ are $\tan\alpha$ and $\tan\beta$. Express $\sec(\alpha+\beta)$ in terms of a, b and c.

6. If $a=x\cos\theta+y\sin\theta$ and $b=x\sin\theta-y\cos\theta$, prove that $\tan\theta=\frac{bx+ay}{ax-by}$.

7. Solve $3\tan^3 x-3\tan^2 x=\tan x-1$ for $0\le x\le\pi$

8. Prove that $\cos^5 x = \frac{\cos 5x+5\cos 3x+10\cos x}{16}$

9. Express $10\sin x\cos x+12\cos 2x$ in the form $R\sin(2x+\alpha)$. Hence solve the equation $10\sin x\cos x+12\cos 2x=-7$ for $0^\circ \le x \le 360^\circ$

10. Prove that if $A+B+C=180^\circ$ then $\cos^2 2A+\cos^2 2B+\cos^2 2C = 1+2\cos 2A\cos 2B\cos 2B$